RESEARCH ARTICLE

Duty cycle allocation to maximize network lifetime of wireless sensor networks with delay constraints

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ABSTRACT

In wireless sensor networks, the routing control overhead could be large because multiple relays are involved in the routing operation. In order to mitigate this problem, a promising solution is to use tier-based anycast protocols. The main shortcoming of these protocols is that they can consume a much greater amount of energy as compared with other competing protocols using deterministic routing. In this paper, we analyze, in depth, a tier-based anycast protocol and develop a new technique of improving network lifetime. Our solution is guided by our analytic framework that consists of subtiering and a new forwarding protocol called ‘scheduling controlled anycast protocol’. We formulate the problem for finding an optimal duty cycle for each tier with a delay constraint as a minimax optimization problem and find its solution, which we show is unique. From the analytical results, we find that the network lifetime can be significantly extended by allocating a different duty cycle adaptively for each tier under a delay constraint. Through simulations, we verify that our duty cycle control algorithm enhances the network lifetime by approximately 70% in comparison with an optimal homogeneous duty cycle allocation. Copyright © 2012 John Wiley & Sons, Ltd.

KEYWORDS
wireless sensor network; tier-based anycast; duty cycle control; monitoring; network lifetime

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1. INTRODUCTION

Wireless sensor networks (WSNs) can provide for a myriad of different services from environmental monitoring to intrusion detection. They are typically characterized by limited battery capacity, high node density, and unidirectional data transmission [1]. The high node density comes from each node’s short sensing range compared to the transmission range and creates many one-hop neighbors [2,3]. The total traffic rate can be very high because most nodes usually generate periodic data, and the total number of nodes is very large. These characteristics of WSN have prevented many previous routing and media access control (MAC) protocols from being deployed on sensor networks [4,5]. The problems that we need to overcome with the WSN protocols are excessive energy consumption in the routing setup, low scalability because of the control overhead, high packet delay because of the periodic sleeping of each sensor node, high collision probability, bandwidth scarcity, and low reliability in packet delivery.

Many WSN-specific solutions have been proposed to overcome these problems [4,6–10], and the anycast protocol is one of the most promising solutions [2,11]. However, a major drawback of this anycast protocol is that it is unable to control each node’s duty cycle. This problem limits its applicability to a network with a very low traffic rate [11]. A variant of the anycast protocols is the tier-based anycast that works for a network with tiers and forwards each data packet using tier information [12–14]. In tier-based anycast, the entire network is organized into tiers centered around the sink, and data packets are forwarded progressively to tiers closer to the sink. It adopts a cross layer approach that unifies MAC and routing protocols, which results in very low routing control overhead. It is also highly robust because it does not use a deterministic routing path. However, with power-saving nodes that sleep and wake up periodically, this protocol may consume a huge amount of energy because a transmission node (TX node) must take the responsibility of finding when the reception node (RX node) wakes up [12,13,15].
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There are also protocols developed to reduce the TX nodes’ loads by controlling the tier width [12,16], but they have limitations in prolonging the network lifetime. In order to mitigate this defect, some protocols prefer using a low cost path to a randomly selected path [13], and others coordinate the wake-up time of each node [17]. The drawback of these approaches is that they incur additional control message overhead and require each node to keep information about others, which, in turn, weakens the merits of the original tier-based scheme, that is, scalability and robustness. Our previous work suggested the analytical model and simple duty cycle allocation algorithm to solve this problem [18]. However, it failed to obtain the optimal duty cycle to satisfy the maximum delay constraint. Moreover, it is very difficult to support a large network size because single channel MAC becomes the performance bottleneck.

In this paper, we develop an analytical anycast model and new multichannel forwarding protocol called the scheduling controlled anycast protocol (SCAN). It is based on our previous work, but it is extended to consider the maximum end-to-end delay constraint. Because of the multichannel MAC of SCAN, it can be used for large-sized wireless sensor networks that are composed of more than 1000 nodes, such as intrusion detection for military area and environmental or crop monitoring.

Our contributions in this paper are threefold.

1. Our analytical model provides a simple way to analyze the characteristics of the tier-based anycast protocol.
2. On the basis of our analytical model, we describe an optimization problem and find an optimal homogeneous duty cycle that satisfies a delay constraint.
3. SCAN finds an optimal heterogeneous duty cycle for each tier in order to save more energy with a delay constraint.

Initially, we deploy the network with a homogeneous duty cycle, and SCAN gathers the energy consumption rate of each node after some time. On the basis of this information, the network allocates a different duty cycle for each tier in order to increase the network lifetime. The optimal duty cycle for each tier is found by solving a minimax problem under the constraint of a worst case delay. Through simulations, we confirm that our duty cycle allocation algorithm improves the network lifetime considerably. An attractive feature of our protocol is that it preserves the merits of conventional tier-based anycast protocols at the cost of a small extra control overhead.

The rest of this paper is organized as follows. In Section 2, we introduce and briefly describe the SCAN protocol. In Section 3, we analyze the tier-based anycast protocol by using our analytic framework to find an optimal network-wide homogeneous duty cycle. Some numerical results are presented to support our analysis. In Section 4, we formulate the problem of finding different optimal duty cycles for different tiers. Simulation results are presented, which illustrate the performance of our duty cycle allocation algorithm in Section 5. We conclude our paper in Section 6.

2. BASIC PROTOCOL DESCRIPTION

An essential role of a WSN forwarding protocol is to deliver the generated data to the sink node within a delay constraint while meeting the requirement of low energy consumption. The forwarding delay and the amount of energy consumed are affected greatly by the duty cycle. Therefore, efficient duty cycle allocation is critical in a successful network deployment. However, in general, it is very difficult to find an optimal duty cycle. Our proposed SCAN aims at finding an optimal duty cycle and allows it to be automatically controlled. In order to achieve this, SCAN follows three phases: Phase 1: homogeneous duty cycle allocation and tier setup, Phase 2: energy consumption reporting, and Phase 3: heterogeneous duty cycle allocation for each tier.

SCAN maximizes the lifetime of a node that has the minimum lifetime among all the nodes. This lifetime is not the same as the network lifetime, but they are tightly correlated where all the nodes are uniformly distributed. If a node breaks down, its neighbor nodes take over the function of the broken one, in some sense, and accordingly, they start to consume their energy faster than before. If all the neighboring nodes’ energies are depleted, the network will fail to collect data from the region of interest. Because of this, it is reasonable to maximize the lifetime of a node that has the minimum lifetime among all the nodes to maximize the network lifetime.

Duty cycle allocation algorithms are needed for Phases 1 and 3 to extend the network lifetime by minimizing the energy consumption.

2.1. Phase 1: homogeneous duty cycle allocation and tier setup

For the initial tier setup, we rely on existing tier setup algorithms [12,14]. In this phase, all the nodes operate at a high duty cycle mode or wake-up mode when deployed in a monitoring area. Then, the sink node, usually located at the network center, floods a tier setup message or transmits it directly to all the nodes. Because of the lack of space, we only explain flooding-based assignment. Because all nodes do not sleep initially, the sink node can flood the tier ID setup messages into the entire network. The setup message contains a hop-count value from the sink node. Therefore, the initial value in the message that the sink node creates is 0. Whenever each node, except the sink node, receives the

1We also assume that our protocol uses existing techniques for the retier setup.
message, it reads the hop-count value of the message and checks if it is a minimum hop-count value or not. If it is the minimum value, it saves the value or updates the old value, increases the value in the message by 1, and floods it again into the neighbor nodes. After the flooding, the minimum hop-count value of each node is a tier ID for the node, and the node enters a low duty cycle mode to save energy.

The tier setup message contains fields such as initial duty cycle and energy consumption reporting time.

2.2. Phase 2: energy consumption reporting

When each node receives the tier setup message, it sets its timer to the energy consumption reporting time, and it transmits its tier ID and information about energy consumption to the sink node when the timer expires. The information contains the energy consumption rates for packet transmission and reception. We assume that these can be calculated from the duration time at each radio state and the specification [19].

2.3. Phase 3: heterogeneous duty cycle allocation

The packet delivered to the sink node contains a field for the experienced end-to-end delay. In this phase, each relay node, including the source node, needs to update this field when packet forwarding happens. Whenever the sink node receives the data packet from the outermost tier (i.e. the tier that is the farthest from the sink node), it calculates and updates the average end-to-end delay. After the energy consumption reporting time, the sink node knows the maximum energy consumption rate of each tier and the average end-to-end delay for the outermost tier. The sink node then calculates an optimal duty cycle for each tier under the given average end-to-end delay condition for the outermost tier. The heterogeneous duty cycle allocation is completed when these duty cycle values are delivered to every node by flooding.

Even though SCAN uses a centralized duty cycle allocation algorithm, the overhead is negligible because of the following features:

- **Low control overhead**: It requires only one heterogeneous duty cycle allocation, and there is no packet exchange after the is allocation completed.
- **Low energy consumption**: The duty cycle allocation message contains only a duty cycle not for each individual node but for each tier. The amount of the total duty cycle information for all tiers is small enough to be carried by embedding into data or data ack packets. For WSNs, TX nodes consume most of its energy to find the wake-up times of RX nodes, not to transmit actual packets. It means that the increased energy consumption caused by the increased packet size is negligible.

- **Low memory requirement**: Each node should save the received duty cycle allocation message into its memory to forward it to neighbor nodes. Generally, 1 byte can be enough for a duty cycle allocation message for one tier. For example, we need only 20 bytes for 20 tiers. This size is small enough even for WSN nodes with tiny memory sizes.
- **High scalability**: The algorithm is scalable for the network size because the complexity of the running time of the algorithm is \(O(N)\) when the total number of tiers is \(N\).

3. HOMOGENEOUS DUTY CYCLE ALLOCATION

SCAN uses a multichannel MAC protocol. We now explain our network model to show how SCAN works and how a homogeneous duty cycle is initially calculated using our analytical framework [18].

3.1. Network model

Each node is assigned a tier identification number (ID) on the basis of its distance from the sink node, as shown in Figure 1. The width of each tier is set to be equal. Only one sink node exists in the network, and it has a tier ID equal to 0. The tier closest to the sink has a tier ID equal to 1. Each subsequent tier is allocated an ID higher than the previous one, starting from the tier that is closest to the sink until the outermost tier is accounted for [12].

If there are multiple sink nodes in the network, each node obtains multiple tier IDs from multiple sink nodes but chooses the smallest tier ID only. Therefore, the network is divided into several subnetworks, and each subnetwork works independently. Because the duty cycle allocation is also performed for each subnetwork, we can consider the duty cycle allocation for a network with only one sink node even for multiple sink node cases.

Each node is allowed to send data packets to the sink node after being assigned its tier ID and starts to sleep and wake up repeatedly to save energy. The first wake-up time of each node is selected randomly. Each node can receive a packet during the wake-up period only but can transmit whenever necessary. That is, if a node has a packet to send during a sleep period, it wakes up to send it and goes back to sleep after transmission.

A TX node needs to find an RX node first that is closer to the sink node. In most of the previous anycast protocols, the TX node uses short control packets to find the RX node [12,13]. This is inefficient in a network with low duty cycle and high data rate because the channel occupancy is high. We use a different approach to solve this problem, which is similar to Receiver-Initiated asynchronous duty cycle MAC (RI-MAC) [15] where the RX node sends a beacon whenever it wakes up and the TX node listens to the common channel until it hears the beacon. This reduces the channel occupancy greatly and improves the performance of both the delay and the throughput.
If the node density is high and the monitoring interval is short, the collision probability of beacons or data packets becomes higher. To alleviate this problem, we consider an extended RI-MAC as shown in Figure 2.

- Each RX node transmits a beacon over the common control channel if it senses that the channel is idle and switches to a randomly selected data channel to wait for a data packet [20]. It also uses the random back-off algorithm.
  - The beacon contains the RX node’s tier ID and the data channel number.
  - A TX node that hears the beacon switches to the data channel indicated in the beacon, and if it wins the contention, the transmission of a data packet is completed.
When a TX node receives a beacon, it compares its own tier ID with that of the RX node, and it switches to the specified data channel only if its own ID is greater. If the RX node received a data packet successfully, it forwards it to a node with a lower tier ID, that is, closer to the sink node. This means that the RX node becomes a TX node, and this process repeats until the packet reaches the sink node.

We assume that the sink node can listen to all the data channels simultaneously, and it never sleeps. This feature enables the neighboring nodes of the sink node to send data packets without listening to the beacon. Whenever the neighboring nodes have packets destined for the sink node, they directly join the contention in randomly selected data channels without listening to the beacon.

In the forwarding process, the tier width is a critical parameter which affects the overall performance. Let $R$ be the transmission range of a sensor node and $w$ be the width of each tier. Then, $w = R/c$, where the constant $c$ is given as 2.2 for optimality [12].

We also assume that the network works as follows:

- All the nodes are fixed and uniformly distributed with density $\rho$.
- Each node, except the sink node, transmits a data packet to the sink periodically with the interval $1/f$ (s).
- Each node repeats the sleep and wake-up patterns, and the sleep interval is exponentially distributed with mean $1/\lambda_k$ (s) for tier $k$.

We logically divide each tier into multiple subtiers of equal width, and each subtier has a network-wide unique ID. The subtier closest to the sink node has the subtier ID 1, and the ID of each subsequent tier is increased by one. The subtier is only for the analysis of the network such that it is independent of packet forwarding.

Let $N$ denote the number of total tiers and $M$ the number of subtiers in each tier. Let us fix the width of each subtier to be one without loss of generality (Table I).

### Table I. Definition of variables used in the numerical analysis.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
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<tbody>
<tr>
<td>$S_k$</td>
<td>Area of subtier $i$ where nodes can receive packets from a node in subtier $k$.</td>
</tr>
<tr>
<td>$q_k^i$</td>
<td>Probability of subtier $i$ receiving a packet from subtier $k$.</td>
</tr>
<tr>
<td>$G_i$</td>
<td>Total packet transmission rate of subtier $i$.</td>
</tr>
<tr>
<td>$\mu_i$</td>
<td>Average packet transmission rate of a node in subtier $i$.</td>
</tr>
<tr>
<td>$F_i$</td>
<td>Traffic rate generated from the outermost tier and received by subtier $i$.</td>
</tr>
<tr>
<td>$d_i$</td>
<td>Average delay experienced from the outermost tier through subtier $i$.</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Average time taken to exchange packets from the beacon through the data ack.</td>
</tr>
</tbody>
</table>

3Each tier is divided into logical subtiers for analysis, and each subtier within a tier has the same average sleep interval. Because of the random sleep interval, we can effectively avoid a consecutive wake-up time collision between two RX nodes.

### 3.2. Energy consumption rate

Assuming that the nodes in a subtier are homogeneous, we can analyze the average TX and RX energy consumption rates of each node in each subtier on the basis of our previous work [18].

From Figure 1, we obtain

$$
S_k^i \simeq \arccos \left( \frac{i^2 + k^2 - R^2}{2ik} \right) \cdot (2i + 1) \text{ for } i < k
$$

where $S_k = \sum_{i=\max([k-M+1],1)}^{[k/M-1]M} S_k^i$ and $[\cdot]$ is the ceiling operator.

$$
q_k^i = \frac{S_k^i}{S_k}
$$

Then, we obtain

$$
\mu_i = \frac{G_i}{\pi(2i+1)\rho}
$$

Then, we can derive the average channel listening time $W_i$ of a TX node in subtier $i$ waiting to receive an RX beacon by taking the inverse of the sum of the wake-up rates of all the nodes that can receive and forward data packets.

$$
W_i = \left( \frac{[i/M-1]M}{\sum_{h=\left[\frac{i}{M}-1\right]}^{\left[\frac{i}{M}\right]} \rho S_h^i \lambda_h} \right)^{-1} \text{ for } i > cM
$$

and $W_i = 0$ for $i \leq cM$ because the sink node does not sleep at all, and it immediately replies with a data ack to a TX.
node. $\lambda_h$ is the average wake-up rate of a node in subtier $h$ and is set to $\lambda$ in the homogeneous network. From this, we can calculate the average TX energy consumption rate of a node in subtier $i$, $\tau_i$, as

$$\tau_i \simeq \mu_i \left( W_t P_{RX} + E_{RX}^{beacon} + E_{CS}^{data} + E_{TX}^{data} + E_{RX}^{ack} \right)$$

where $P_{RX}$ is the energy consumption rate when the radio is in the RX state, $E_{RX}^{beacon}$ is the energy consumed when a beacon is received from the common channel, $E_{CS}^{data}$ is the average consumed energy during carrier sensing in the contention window, $E_{TX}^{data}$ is the energy consumed when transmitting a data packet, and $E_{RX}^{ack}$ is the energy consumed when receiving a data ack. We can also approximate the average RX energy consumption rate of a node in the subtier $i$, $\sigma_i$, as

$$\sigma_i \simeq \lambda_i \left( E_{CS}^{common} + E_{TX}^{beacon} + E_{IL}^{data} \right)$$

where $E_{CS}^{common}$ is the average consumed energy for carrier sensing in the common channel, $E_{TX}^{beacon}$ is the energy consumed when transmitting a beacon in the common channel, and $E_{IL}^{data}$ is the energy consumed during idle listening when waiting for an incoming data packet.

### 3.3. Optimal homogeneous duty cycle allocation

In WSNs, the average end-to-end delay for all nodes is sometimes used to represent the delay constraint. However, this may not be appropriate for an anycast network because nodes in a tier far from the sink node experience a very long average delay. Therefore, we can define a new delay constraint as follows.

$$\max_{h \in S} \{ D_h \} \simeq D_{N-1} \leq D_{MAX}$$

where $D_{MAX}$ is the maximum allowed delay, $D_h$ is the average end-to-end delay of nodes in tier $h$, and $S$ is the set of all the tiers, $\{1, 2, \ldots, N-1\}$.

We use the maximum of average delays instead of the maximum delay because it is simple and generally used in environmental monitoring [11]. However, our duty cycle allocation algorithm can be applied in the WSNs when the maximum delay is important. The main purpose of the proposed heterogeneous duty cycle allocation algorithm is to decrease energy consumption without increasing the maximum average delay. Because the maximum delay is linearly proportional to the maximum average delay, we can use the proposed heterogeneous duty cycle allocation algorithm with the worst case delay without significant modification.

To calculate $D_{N-1}$, we should find $d_i$ and $F_i$ first. $d_i$ is given as

$$d_i = \frac{\sum_{k=X_i}^{Y_i} d_k F_k q_k^i}{\sum_{k=X_i}^{Y_i} F_k q_k^i} + W_i + \delta$$

and $F_i$ is

$$F_i = \sum_{k=X_i}^{Y_i} F_k q_k^i \text{ for } i < k$$

From these results, we can calculate $D_{N-1}$ as

$$D_{N-1} = \frac{\sum_{k=1}^{cM} d_i F_i}{\sum_{k=1}^{cM} F_i}$$

We now show how the duty cycle affects the energy consumption. The duty cycle of each node in subtier $i$ is $\phi_i$, and the wake-up period is fixed at $T_w$. We then obtain

$$\phi_i = \frac{T_w}{\lambda_i^{-1}} = \lambda_i T_w$$

Note that in the homogeneous setting, $\lambda_i = \lambda$ and $\phi_i = \phi, \forall i \in S$. From Equations (7), (11), and (13), $W_t, d_i$ and $D_{N-1}$ increase monotonically if $\phi$ decreases. As a result, we can find the minimum $\phi$ that satisfies the delay constraint (10). Also, from Equations (8) and (9), $\tau_i$ decreases as $\phi$ decreases.

The problem can be formulated as

$$\text{P1 : minimize } \max_{i \in S} \{ \tau_i + \sigma_i \}$$

subject to $D_{N-1} \leq D_{MAX}$

To solve this problem, we consider

$$H = \text{argmax}_{i \in S} \{ \tau_i + \sigma_i \}$$

§For example, we measured the maximum average delay, maximum delay and the standard deviation of maximum delay, respectively. For previous and proposed duty cycle allocations, we obtained 1835, 6955 and 708, and 1840.6, 7429 and 714.8, respectively, when the unit is slot time. Therefore, we can say that our proposed algorithm allocates heterogeneous duty cycle for each tier with very little delay increment for both maximum average and worst case delay cases.
Algorithm 1 Calculate $\phi^*$ for P1

1: Find $\phi_1$ s.t. $D_{N-1} = D_{MAX}$
2: Find $H$ s.t. $\tau_H + \sigma_H \geq \tau_i + \sigma_i, \forall i \in S$
3: Find $\phi_2(>0)$ s.t. $\frac{\partial (\tau_H + \sigma_H)}{\partial \phi} = 0$
4: $\phi^* \leftarrow \max\{\phi_1, \phi_2\}$

$H$ is dependent on $M$ and $N$ but not on $\phi$ because $\mu_i$ is independent of $\phi$, that is,

$$\max_{i \in S}\{\tau_i + \sigma_i\}$$

$$= \tau_H + \sigma_H$$

$$= \mu_H \cdot (W_H P_{RX} + E_1) + \lambda E_2$$

$$= \frac{\mu_H \cdot \left( \frac{T_w}{\sum_{h=1}^{H-1} M h^2 \cdot \phi} \cdot P_{RX} + E_1 \right) + \frac{E_2 \phi}{T_w}}{\sum_{h=1}^{H-1} M h^2 \cdot \phi}$$

where $E_1 = E_{beacon} + E_{data} + E_{ack} + E_{RX}$ and $E_2 = E_{common} + E_{TX} + E_{data}$.

From Equations (13), (14), (16), and (17), we can solve P1. The procedures are summarized in Algorithm 1.

4. HETEROGENEOUS DUTY CYCLE ALLOCATION

In reality, each tier has a different energy consumption rate, so using the homogeneous duty cycle will result in a highly inefficient solution to prolonging the network lifetime. In this section, we consider a heterogeneous duty cycle allocation algorithm that aims at finding an optimal duty cycle for each tier under a delay constraint. The solution uses information about the RX and TX measured energy consumption rates and the measured delay. This is different than the requirement for the homogeneous duty cycle allocation, which also needs the energy consumption rate of each radio state and the average time for the contention window, the packet transmission time, etc. This feature enables us to use our algorithms without modifying them on the basis of different environments such as network topology or radio chipsets. To collect information about the delay and energy consumption, each node inserts the end-to-end delay field into data packets generated by itself and creates the energy consumption reporting message during the energy consumption reporting phase.

4.1. Adjustment of optimal homogeneous duty cycle

The sink node initially calculates an optimal homogeneous duty cycle for the given network in the first phase using some assumptions about network topology and node density. These assumptions fail to capture the true optimal solution. Then, when the sink node collects the complete network information, it finds an actual optimal homogeneous duty cycle, and it is internally used to get an optimal heterogeneous duty cycle. To avoid confusion, we define $\bar{x}$ as the value obtained from the measurement and $x$ as the value from the numerical model.

$$P2:$$

$$= \min_{\phi} \max_{i \in S}\{\bar{\tau}_i + \bar{\sigma}_i\}$$

subject to $\bar{D}_{N-1} \leq D_{MAX}$

From Equation (17), we can assume that $E_1 = 0$ because $\delta \ll W_i$ and $W_i \cdot P_{RX} \ll E_1$ for a general any-cast network. From these assumptions and Equations (11), (13), and (17), we can obtain

$$\bar{\tau}_H \propto \frac{1}{\phi^*}, \quad \bar{D}_{N-1} \propto \frac{1}{\phi^*}, \quad \bar{\sigma}_H \propto \phi^*$$

(19)

where $\bar{H} = \arg\max_{i \in S}\{\bar{\tau}_i + \bar{\sigma}_i\}$. We first express the ratio of the outermost tier delay to the maximum allowed delay as $r = \frac{\bar{D}_{MAX}}{\bar{D}_{N-1}}$. Then, the duty cycle can be given as

$$\tilde{\phi}_1 = r \cdot \tilde{\phi}^*$$

(20)

If no delay constraint exists, the energy consumption of a tier that consumes the highest energy among the $M$ tiers is minimized when the duty cycle is set to

$$\phi_2 = \sqrt{(\bar{\tau}_H \tilde{\phi}^*/ (\bar{\sigma}_H / \tilde{\phi}^*))}$$

(21)

As a result, when the delay constraint is given, the duty cycle should be set to

$$\tilde{\phi}^* = \max\{\tilde{\phi}_1, \tilde{\phi}_2\}$$

(22)

Let $\bar{D}_{N-1}^*$ denote the new average experienced delay from the outermost tier to the sink node after allocating $\tilde{\phi}^*$. Then, we obtain

$$\bar{D}_{N-1}^* \approx \frac{\tilde{\phi}^*}{\phi^*} \cdot \bar{D}_{N-1}$$

(23)
respectively. We observe some important features related to inter-tier dependency from our numerical model and results in Figures 3 and 4.

To explain the characteristics of a tier-based anycast protocol, we now describe the numerical analysis results. Figure 3 shows the average packet reception probability at tier \( n-1 \) through \( n-3 \) when the TX node is selected randomly from tier \( n \), labeled by Average. In case that the TX node consumes the highest amount of energy, one in tier \( n \) is labeled as Critical node\(^3\).

Figure 4 shows the simulation results and the numerical results obtained from the analysis of the energy consumption of a critical node in each subtier for data transmission, data reception, and the total in a homogeneous circular network. Total RX shows the total energy consumption in data packet reception, beacon and data ack exchange, carrier sensing, and additional overhead caused by collision and random back-off. Total TX is the total energy consumption in transmitting data packets and the contingent overhead. Total represents the sum of the Total RX and Total TX. Sim and Num stand for simulation results and analysis results, respectively. We observe some important features related to inter-tier dependency from our numerical model and results in Figures 3 and 4.

4.2. Optimal heterogeneous duty cycle allocation

To reduce the transmission energy of a node, the duty cycle of each node in the receiving tier should be increased according to Equation (8). However, this may decrease the network lifetime because nodes in the receiving tier use more energy according to Equation (9). On the other hand, if we decrease the duty cycle of a node to reduce the reception energy, the node in the transmitting tier needs to spend a longer time listening to the wake-up beacon, which leads to more energy consumption. Therefore, to choose an appropriate duty cycle at each tier, we need to carefully consider inter-tier dependency of the energy consumption.

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\(^3\)Note that critical nodes in tier \( n-3 \) can not receive any packet from tier \( n \) when \( c = 2.2 \).
Figure 3. So, if we want to extend the lifetime of a network with \( N \) tiers, the problem can be formulated as follows. For the given constants \( t_3, t_4, \ldots, t_{N-1} \) \((t_i \geq 0 \text{ for } 3 \leq i \leq N-1)\)

\[
\text{P3: minimize } \quad g(\bar{a})
\]

where \( g(\bar{a}) = \max \left\{ \alpha_1, \alpha_2, \alpha_3 + \frac{t_3}{\alpha_2}, \ldots, \alpha_{N-2} + \frac{t_{N-2}}{\alpha_{N-3}} \right\} \)

subject to \( \sum_{i=2}^{N-2} \frac{1}{\alpha_i} \leq N-3 \), \( \alpha_i > 0 \) for \( 1 \leq i \leq N-2 \)

Each term in the \( \max \{ \} \) operator is the normalized total energy consumption, and the constraint means that the average end-to-end delay of tier \( N-1 \) should not be increased when the duty cycle of each tier varies to meet the delay constraint (10).

Before solving P3, we consider the next problem of P4, which removes the last term, \( \frac{t_{N-2}}{\alpha_{N-3}} \), and replaces the inequality constraint with the equality from P3.

\[
\text{P4: minimize } \quad \hat{g}(\bar{a})
\]

where \( \hat{g}(\bar{a}) = \max \left\{ \alpha_1, \alpha_2, \alpha_3 + \frac{t_3}{\alpha_2}, \ldots, \alpha_{N-2} + \frac{t_{N-2}}{\alpha_{N-3}} \right\} \)

subject to \( \sum_{i=2}^{N-2} \frac{1}{\alpha_i} = K \) \((\leq N-3)\), \( \alpha_i > 0 \) for \( 1 \leq i \leq N-2 \)

We now show that the optimal solution to P4 is obtained when all the terms in the \( \max \{ \} \) operator of the objective function are equal.

**Proposition 1.** If there exists a vector \( \alpha^* = (\alpha_1^*, \alpha_2^*, \ldots, \alpha_{N-2}^*) \) that satisfies the constraints of P4 and

\[
\alpha_1^* = \alpha_2^* = \alpha_3^* + \frac{t_3}{\alpha_2^*} = \alpha_4^* + \frac{t_4}{\alpha_3^*} = \ldots = \alpha_{N-2}^* + \frac{t_{N-2}}{\alpha_{N-3}^*} \quad (27)
\]

then \( \alpha^* \) is an optimal solution to P4.

Proposition 1 gives a key property of the optimal solution, but we do not know yet whether such a vector \( \alpha^* \) exists. Proposition 2 shows that \( \alpha^* \) always exists, and it is unique.

**Proposition 2.** The vector \( \alpha^* \) exists, and it is unique.

Algorithm 2 presents the pseudocode to find an optimal solution to P4. We denote \( K \) for \( K' \) by P4\((K')\).

**Proposition 3.** \( \hat{g}(\bar{a}^*(K_1)) \geq \hat{g}(\bar{a}^*(K_2)) \) and \( \alpha_i^*(K_1) \geq \alpha_i^*(K_2), \forall i, \) if \( \alpha_i^*(K_1) = (\alpha_1^*(K_1), \ldots, \alpha_{N-2}^*(K_1)) \) and \( \bar{a}^*(K_2) = (\alpha_1^*(K_2), \ldots, \alpha_{N-2}^*(K_2)) \) are the optimal solutions to P4\((K_1)\) and P4\((K_2)\), respectively, and \( 0 < K_1 \leq K_2 \leq N-3 \).

We now solve P3 using these results.

**Proposition 4.** If \( \tilde{\alpha}^* = (\tilde{\alpha}_1^*, \tilde{\alpha}_2^*, \ldots, \tilde{\alpha}_{N-2}^*) \) exists and satisfies the constraints of P3, and

\[
0 < \tilde{\alpha}_1^* = \tilde{\alpha}_2^* = \tilde{\alpha}_3^* + \frac{t_3}{\tilde{\alpha}_2^*} = \ldots = \tilde{\alpha}_{N-2}^* + \frac{t_{N-2}}{\tilde{\alpha}_{N-3}^*} = \frac{t_{N-1}}{\tilde{\alpha}_{N-2}^*} \quad (28)
\]

then \( \tilde{\alpha}^* \) becomes an optimal solution to P3.

Although Proposition 4 is similar to Proposition 1, we can not say that P3 always has an optimal solution. The proofs of Propositions 1 through 4 are given in the Appendix.

**Lemma 1.** If \( \tilde{\alpha}^* = (\tilde{\alpha}_1^*, \tilde{\alpha}_2^*, \ldots, \tilde{\alpha}_{N-2}^*) \) is the optimal solution to P4\((N-3)\) and satisfies \( \hat{g}(\alpha^*) \geq \frac{t_{N-1}}{\tilde{\alpha}_{N-2}^*} \), then \( \tilde{\alpha}^* \) is an optimal solution of P3.
Algorithm 3 Calculating $\alpha_i^*$ for $P3$

1: $K_L = \varepsilon_1$, $K_R = N - 3$
2: if $\hat{g}(\alpha^*(N - 3)) < \frac{t_{N-1}}{\sigma_{N-2}^2(N-3)}$ then
3: loop
4: $K \leftarrow \frac{K_L + K_R}{2}$
5: Solve $P_4(K)$
6: if $|\hat{g}(\alpha^*(K)) - \frac{t_{N-1}}{\sigma_{N-2}^2(K)}| < \varepsilon_2$ then
7: Return $\alpha_i^*$
8: else if $\hat{g}(\alpha^*(K)) < \frac{t_{N-1}}{\sigma_{N-2}^2(K)}$ then
9: $K_R \leftarrow K$
10: else if $\hat{g}(\alpha^*(K)) > \frac{t_{N-1}}{\sigma_{N-2}^2(K)}$ then
11: $K_L \leftarrow K$
12: end if
13: end loop
14: else
15: Return $\alpha_i^*$
16: end if

From Lemma 1, we also find an optimal solution to $P3$ when the optimal solution to $P4(N - 3)$ satisfies $\hat{g}(\alpha^*) \geq \frac{t_{N-1}}{\sigma_{N-2}^2}$. We now consider that the optimal solution to $P4(N - 3)$ satisfies $\hat{g}(\alpha^*) < \frac{t_{N-1}}{\sigma_{N-2}^2}$.

Lemma 2. If $\alpha^* = (\alpha_1^*, \ldots, \alpha_{N-2}^*)$ is an optimal solution to $P4(N - 3)$ and satisfies $\hat{g}(\alpha^*) < \frac{t_{N-1}}{\sigma_{N-2}^2}$, $K^*$ (0 < $K^* < N - 3$) always uniquely exists and $\alpha^*(K^*) = (\alpha_1^*(K^*), \ldots, \alpha_{N-2}^*(K^*))$ is the optimal solution to $P3$ as well as $P4(K^*)$.

The proofs of Lemmas 1 and 2 are given in the Appendix. The pseudocode in Algorithm 3 summarizes the procedures in solving $P3$ according to Lemmas 1 and 2. In the pseudocode, we assume that $\varepsilon_1$ and $\varepsilon_2$ are some very small positive constants.

5. PERFORMANCE EVALUATION

We compare our analytical results with simulations to evaluate the efficacy of our analytical model. We use the NS-2 simulator and two-ray propagation model for the channel condition **. We use a random topology of 12 tiers and 1608 nodes in Figure 5 to test our duty cycle allocation algorithms. We compare the heterogeneous allocation results with the homogeneous allocation results in terms of energy consumption, delay at the outermost tier, and packet drop. To better understand the performance, we also vary the total number of channels being used, from 1 through 16. For all the cases, we use one control channel. When there is only one channel, the control channel and the data channel have the same channel ID so that it works as RI-MAC. Considering that most of the environmental monitoring applications requires monitoring intervals from 1 min to 12 h, the packet transmission interval is set to 10 min (0.1 packets/min) in all simulations [21,22]. The tolerable delay constraint is 1 s, and the total simulation

**In anycast, the two-ray propagation model is widely used in simulations. Because of multiple RX nodes and the short RX-TX distance, better channel modeling does not show better results.
time is 1000 min. The energy consumption according to the radio state follows the CC2420 specification [19].

5.1. Evaluation of homogeneous duty cycle allocation

Figure 6(a and b) shows the energy consumption of each tier when the total number of channels is 1 and 16, respectively ††. The RX overhead shows the total energy consumed for beacon, carrier sensing and data ack exchange, and additional overhead due to collision. The total RX shows the total energy consumed when receiving data packets including the RX overhead. The TX overhead is the total energy consumed in carrier sensing and overhead due to collision and random back-off. The total TX is the total energy consumed in transmitting data packets including TX overhead. The total is given as the sum of the total RX and total TX.

In Figure 6(a and b), we observed that the total RX is almost constant and is independent of the total number of channels. The RX overhead, on the other hand, decreases by almost 10% when the total number of channels is 16. From this result, we found that the common control channel is heavily overloaded, and each node fails to send a beacon easily when the total number of channels is 1.

The total TX with 1 channel is higher by 33% compared to that with 16 channels. Because the TX overhead is very small and almost independent of the total number of channels, we can conclude that an increased beacon interval also increases the total TX.

Because the RX overhead and the TX overhead decrease with the tier ID for all the cases, we can estimate that the common control channel closer to the sink node becomes more congested. Although this control channel is used by packet reception and transmission simultaneously, the RX overhead is larger than the TX overhead for all the tier IDs, regardless of the total number of channels. This means that the busy control channel leads to more packet reception failure than packet transmission.

In anycast protocols, the TX overhead is incurred when the TX node fails to receive a beacon packet in the control channel, or the TX node senses a carrier in the data channel. We can ignore the overhead from the data channel because of the low channel occupancy. The TX node can receive a beacon packet from any candidate nodes, and the probability of having failure in the control channel is quite low. Consequently, the TX overhead becomes smaller than the RX overhead. From Figure 6(a and b), we observe that the network lifetime increases by approximately 20% with the total number of data channels. We will discuss the relation between the network lifetime and the total number of channels in Section 5.2.

5.2. Evaluation of heterogeneous duty cycle allocation

Figure 7(a and b) shows the energy consumption and the allocated duty cycle of each tier when the total number of channels is 1 and 16, respectively. For better comparison, we show the relative duty cycle, that is, the ratio of the allocated heterogeneous duty cycle to the previous homogeneous duty cycle. Our heterogeneous duty cycle allocation finds the optimal duty cycle for each tier for all cases even though the RX overhead is increased. The RX overhead can cause errors in duty cycle allocation because it is not considered in the allocation. However, in Figure 7(a and b), the RX overhead is almost proportional to the total RX, and this property avoids the error in the duty cycle allocation.

Surprisingly, it is not optimal to allocate a higher duty cycle to a tier closer to the sink node. We can obtain better performance by allocating the duty cycle following a convex shape against the tier ID, as shown in Figure 7(a and b).

††We omit results for the other number of channels because of the lack of space.
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Figure 7. Comparison of the energy consumptions according to the total number of channels when heterogeneous duty cycle allocation is used.

Figure 8. Comparison of the network lifetime according to the total number of channels in the random topology.

Figure 9. Comparison of the delay and drop ratio according to the total number of channels in the random topology.

Figure 10. Random topology of 18 tiers, 1608 nodes, and a sink node at the edge of the network.

Figure 8 shows the simulation results for the network lifetime according to total number of channels. The heterogeneous allocation increases the network lifetime by 40% compared to the homogeneous allocation, regardless of the number of channels. In addition, the energy consumption decreases with the number of channels, and the heterogeneous allocation with 16 channels increases the network lifetime by 70% compared to the homogeneous allocation with one channel.

Figure 9 shows that the average delay of the outermost tier drops rapidly when the number of channels increases from 1 to 2. Then, it is about the same when the number of channels is greater than two. From these results, we can find that the common channel is the bottleneck in delay performance. For all the cases, we can decrease the delay by at least 10% if we use the heterogeneous allocation. The number of dropped packets also decreases with the number of channels.

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5.3. Relation between duty cycle allocations and the location of a sink node

The homogeneous duty cycle allocation algorithm finds the optimal duty cycle under the assumptions such as the circular network and the sink node at its center. Because of these tight assumptions, the result can be deteriorated in the real world. However, the heterogeneous duty cycle allocation algorithm finds its duty cycle on the basis of the measured data for energy consumption and end-to-end delay. This feature overcomes the restriction of the homogeneous duty cycle allocation algorithm. We conducted another simulation to verify it. In this simulation, we moved the sink node from the center to the bottom of the network, and the total number of tiers increases to 18 from 12. The tier configuration for each node is shown in Figure 10. We used 16 channels, and other simulation parameters are the same as the previous simulation.

Simulation results are shown in Figure 11. We can see that we obtained a homogeneous duty cycle that is too low in Figure 11(a). As the number of tiers is increased, the relayed data rate is also increased, and it results in a higher RX overhead. However, the heterogeneous duty cycle allocation algorithm adjusted each duty cycle for each tier and minimized the maximum energy consumption effectively, as shown in Figure 11(b). From this result, we can verify that our heterogeneous duty cycle allocation can be adopted for real WSNs.

6. CONCLUSION

In a tier-based anycast wireless sensor network (WSN), allocating the duty cycle is a critical problem that impacts the network lifetime. However, finding an optimal duty cycle is very difficult in general. In this paper, we first proposed an optimal homogeneous duty cycle allocation algorithm and a new protocol called SCAN. SCAN uses multichannels and an optimal heterogeneous duty cycle allocation. Through a numerical analysis and simulation, we showed that our considered SCAN increases the network lifetime by approximately 70% compared to the tier-based scheme that uses our optimal homogeneous duty cycle value with one channel. The performance gap becomes much larger when the optimal homogeneous duty cycle is not used. SCAN can support a large-sized network of more than 1000 nodes that requires a tight delay bound. Another merit of SCAN is that it is able to control the duty cycle for each tier, adaptively. Contrary to our intuition, we found that the best performance was not achieved by allocating a higher duty cycle to a tier closer to the sink node.

Another attractive feature of our duty cycle allocation algorithm is that it preserves the merits of the original tier-based anycast protocols and achieve much longer network lifetimes. Wireless monitoring is a very important application for WSNs. However, existing WSN protocols do not meet the long network lifetime property and the tight delay constraints, simultaneously. We hope our research results will be a practical solution in the application area.

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**APPENDIX**

**Proof of proposition 1**

Suppose a vector $\beta \triangleq (\beta_1, \beta_2, \beta_3, \cdots, \beta_{N-2})$ satisfies the constraint of $P4$. If $\beta$ gives a smaller value of the objective
function than $\alpha^*$ does, the following holds.

$$\beta_1 < \alpha^*_1, \beta_2 < \alpha^*_2, \beta_3 + \frac{t_3}{\beta^2_2} < \alpha^*_3 + \frac{t_3}{\alpha^2_2}, \cdots.$$  

$$\beta_{N-2} + \frac{t_{N-2}}{\beta_{N-3}} < \alpha^*_N + \frac{t_{N-2}}{\alpha^2_{N-3}}$$

From this, we obtain

$$\beta_1 < \alpha^*_1, \beta_2 < \alpha^*_2, \beta_3 - \alpha^*_3 < t_3 \frac{1}{\alpha^2_2} - \frac{1}{\beta^2_2} < 0, \cdots,$$

$$\beta_{N-2} - \alpha^*_N < t_{N-2} \left( \frac{1}{\alpha^2_{N-3}} - \frac{1}{\beta^2_{N-3}} \right) < 0$$

That is, $0 < \beta_i < \alpha^*_i, \forall i$.

But, this contradicts the assumption that $\tilde{\beta}$ satisfies the constraint of $P_4$ because

$$\sum_{i=2}^{N-2} \frac{1}{\beta_i} > \sum_{i=2}^{N-2} \frac{1}{\alpha^*_i} = K$$

Therefore, such a vector cannot exist, and we can conclude that $\alpha^*$ is an optimal solution to $P_4$.

**Proof of proposition 2**

Let us define $f_i(x)$ as

$$f_i(x) = \begin{cases} 
  x & \text{for } x \geq 0, \text{ if } i = 1 \text{ and } 2, \\
  x - \frac{t_i}{f_{i-1}(x)} & \text{for } x > z_{i-1}, \text{ otherwise} 
\end{cases}$$

where $z_{i-1}$ is the zero of $f_{i-1}(x)$. Then, $\frac{df_i}{dx} > 0$ for any point in the domain. In addition, because $\lim_{x \downarrow z_{i-1}} f_i(x) \rightarrow -\infty, \lim_{x \rightarrow \infty} f_i(x) \rightarrow \infty$, and $f_i(x)$ is continuous and increases monotonically, the zeros of $f_i(x)$ satisfy $z_{N-1} > z_{N-2} > \cdots > z_2 = z_1 = 0$.

Let us define the function $f(x) = \sum_{i=2}^{N-2} \frac{1}{f_i(x)}$ for $x > z_{N-2}$. $f(x)$ decreases monotonically because $f_i(x), \forall i$, is always greater than 0 and increases monotonically. Then, we have

$$\lim_{x \downarrow z_{N-2}} f(x) = \lim_{x \downarrow z_{N-2}} \sum_{i=2}^{N-2} \frac{1}{f_i(x)} \rightarrow \infty,$$

$$\lim_{x \rightarrow \infty} f(x) = \lim_{x \rightarrow \infty} \sum_{i=2}^{N-2} \frac{1}{f_i(x)} = 0 \quad (5)$$

In summary,

i) $\lim_{x \downarrow z_{N-2}} f(x) \rightarrow \infty (> K)$.

ii) $\lim_{x \rightarrow \infty} f(x) = 0 (< K)$.

iii) $f(x)$ is a continuous and monotonically decreasing function.

Therefore, there exists $x^* (z_{N-2})$ such that $f(x^*) = K$, and $x^*$ is unique.

Because $x^*$ uniquely exists, so does $f_i(x^*)$, $\forall i$. Defining $\alpha^*_i = f_i(x^*)$, we have $\alpha^*_i = x^*$ for $i = 1$ and 2, and

$$\alpha^*_i = f_i(x^*) = x^* - \frac{t_i}{f_{i-1}(x^*)} = x^* - \frac{t_i}{\alpha^*_i-1} \quad (6)$$

for $3 \leq i \leq N-2$. This indicates that $x^* = \alpha^*_1 + \frac{t_1}{\alpha^*_1-1}$.

Therefore, we obtain

$$x^* = \alpha^*_1 = \alpha^*_2 = \alpha^*_3 + \frac{t_3}{\alpha^*_2} = \cdots = \alpha^*_N - 2 + \frac{t_{N-2}}{\alpha^*_N-3} \quad (7)$$

and $\tilde{\alpha}^* (\alpha^*_1, \cdots, \alpha^*_N-2)$ satisfies the property of Proposition 1.

**Proof of proposition 3**

Defining $x^*_1 = f^{-1}(K_1)$ and $x^*_2 = f^{-1}(K_2)$, we obtain $x^*_1 \geq x^*_2$ because $f(x)$ decreases monotonically.

From this, we know

$$x^*_1 = \alpha^*_1 = \alpha^*_2 = \alpha^*_3 + \frac{t_3}{\alpha^*_2} = \cdots = \alpha^*_N - 2 + \frac{t_{N-2}}{\alpha^*_N-3} \quad (8)$$

and, that is, $g(\tilde{\alpha}^*(K_1)) \geq g(\tilde{\alpha}^*(K_2))$.

$$\alpha^*_1(K_1) \geq \alpha^*_1(K_2), \alpha^*_2(K_1) \geq \alpha^*_2(K_2),$$

$$\alpha^*_3(K_1) - \alpha^*_3(K_2) \geq t_3 \frac{1}{\alpha^2_2(K_2)} - \frac{1}{\alpha^2_2(K_1)} \geq 0, \cdots,$$

$$\alpha^*_N - 2(K_1) - \alpha^*_N - 2(K_2) \geq t_{N-2} \times \frac{1}{\alpha^2_{N-3}(K_2)} - \frac{1}{\alpha^2_{N-3}(K_1)} \geq 0 \quad (9)$$

Thus, we obtain $\alpha^*_i(K_1) \geq \alpha^*_i(K_2), \forall i$.

**Proof of proposition 4**

Suppose another $\tilde{\beta} \triangleq (\beta_1, \beta_2, \cdots, \beta_{N-2})$ satisfies $P_3$ and $g(\tilde{\beta}) < g(\tilde{\alpha}^*)$. Therefore,

$$\beta_1 < \alpha^*_1, \beta_2 < \alpha^*_2, \beta_3 + \frac{t_3}{\beta^2_2} < \alpha^*_3 + \frac{t_3}{\alpha^2_2}, \cdots,$$

$$\beta_{N-2} + \frac{t_{N-2}}{\beta_{N-3}} < \alpha^*_N - 2 + \frac{t_{N-2}}{\alpha^*_N-3}, \beta_{N-2} < \frac{t_{N-2}}{\alpha^*_N-3} \quad (10)$$
From this, we can obtain
\[ \beta_1 < \alpha_1, \beta_2 < \alpha_2, \beta_3 - \alpha_3 < t_3 \left( \frac{1}{\alpha_2} - \frac{1}{\beta_2} \right) < 0, \cdots, \]
\[ \beta_{N-2} - \alpha_{N-2} < t_{N-2} \left( \frac{1}{\alpha_{N-3}} - \frac{1}{\beta_{N-3}} \right) < 0, \forall \]
\[ \times \left( \frac{1}{\alpha_{N-2}} - \frac{1}{\beta_{N-2}} \right) > 0 \tag{11} \]

This is a contradiction because these are \( \beta_{N-2} < \alpha_{N-2} \) and \( \beta_{N-2} > \alpha_{N-2} \). Finally, \( \hat{\beta} \) can not exist, and \( \alpha^* \) is an optimal solution to \( \text{P3} \).

**Proof of lemma 1**

According to the given condition,
\[ g(\alpha^*) = \max \left\{ \hat{g}(\alpha^*), \frac{t_{N-1}}{\alpha_{N-2}} \right\} = \hat{g}(\alpha^*) \tag{12} \]
and assuming that \( \hat{\beta} = (\beta_1, \cdots, \beta_{N-2}) \) satisfies \( \text{P3} \)'s constraints, \( \alpha^* \) is an optimal solution of \( \text{P3} \) because
\[ g(\hat{\beta}) = \max \left\{ \hat{g}(\hat{\beta}), \frac{t_{N-1}}{\beta_{N-2}} \right\} \geq \hat{g}(\hat{\beta}) \tag{13} \]
(13)
\[ \geq \hat{g}(\alpha^*) \text{ (from Proposition 3)} \]

**Proof of lemma 2**

Let us denote an optimal solution to \( \text{P4}(N - 3) \) by \( \alpha^*(K^*) = (\alpha_1^*(K^*), \cdots, \alpha_{N-2}^*(K^*)) \). \( \hat{g}(\alpha^*(K)) \) is a continuous function because \( \hat{g}(\cdot) \) and \( \alpha_1^*(\cdot), \forall i \), are continuous. \( t_{N-1} / \alpha_{N-2}(K) \) is also a continuous function because \( \alpha_{N-2}(K) \) is continuous.

When \( h(K) \equiv \hat{g}(\alpha^*(K)) - \frac{t_{N-1}}{\alpha_{N-2}(K)} \), it is continuous and monotonically decreasing because \( \hat{g}(\alpha^*(K)) \) is a monotonically decreasing function according to Proposition 3. It also satisfies \( \lim_{K \to 0^+} h(K) = \infty \) and \( \lim_{K \to N} h(K) = 0 \). From these, \( K^* (0 < K^* < N - 3) \), which satisfies \( h(K^*) = 0 \), uniquely exists. Then, \( \alpha^*(K^*) \), the solution to \( \text{P4}(K^*) \), is also the optimal solution to \( \text{P3} \) according to Proposition 4.

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